

Tying the Knot

(Knotted Bracelet)

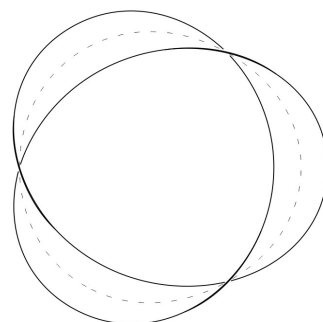
George Hart and Elisabeth Heathfield



Our G4G-13 exchange item is a knotted rubber band that can be worn as a bracelet. People who see you wearing it should be curious how it was made, because it is one continuous loop of rubber band in the form of a trefoil knot (a simple overhand knot). It began its life as an ordinary circular rubber O-ring without a knot, i.e., an unknot. We converted it into a loop with a knot. What we did not do is: cut it, tie a knot, and rejoin it; that would be too easy. And yet there is a well established theorem of knot theory that claims one can not convert an unknot into a trefoil knot without cutting it. So how was an ordinary hardware-store O-ring transformed from being un-knotted to being knotted?

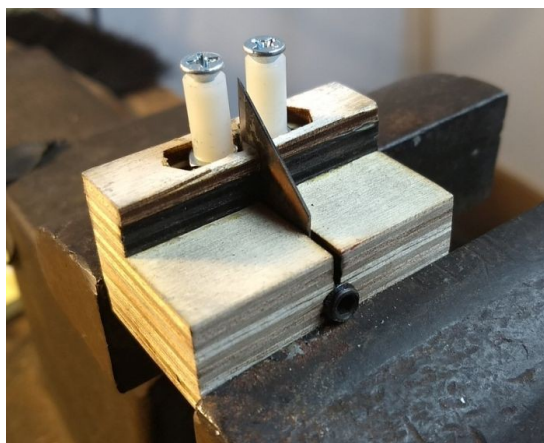
The secret is to make a long spiral slice that twists three-halves of a revolution as it circles the length of the loop.

The O-ring's circular cross-section is reduced to a D-shaped cross section and the circumference of the loop is doubled. The process is analogous to making a three-halves-twist paper Mobius strip and cutting it along the center line, shown dashed at right. The result, as Martin Gardner described in an early column, is that the sliced strip is found to be tied in a trefoil knot. The same thing happens when one starts with a solid ring or a bagel and makes the analogous three-halves-twist spiral cut. Instructions for cutting a bagel into a trefoil knot are available at <http://georgehart.com/bagel/knot.html>



Slicing a rubber O-ring the long way with a three-halves twist requires some precision, but is a fun accomplishment. Do some bagels first, for practice. The process is demonstrated in Hart's video, *Knot Possible?*, which can be seen at

<http://www.simonsfoundation.org/2013/01/03/mathematical-impressions-knot-possible/>



It took several attempts to successfully make the cut shown in that video. Try it and you'll see that if the blade wanders too far from the center-line, it will slip over to the edge of the band and ruin the attempt. Making hundreds of knotted bracelets, so they could be given out as a G4G gift item, required first designing a blade guide. The photo at left shows how its laser-cut wood body holds two nylon tubes that serve as roller bearings, using adjustable bolts. Between them is a utility-knife blade locked in place with a set screw. The arrangement is firmly held in a vice and serves to keep the blade stiffly centered in the O-ring. Guide marks are first drawn on the ring (with a silver-colored marker) to outline the spiral path. Then it is not difficult to twist the rubber to follow the marks as it is pulled into the blade. With practice, a complete cut takes less than a minute.

These G4G-13 knotted bracelets commemorate our “tying the knot” together on March 14, 2018, just three weeks before the conference. Please wear them in celebration with us!